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n which

$$\left.\begin{aligned} t &= r' - r_{h \doteq 0} \\ t' &= r'' - r'_{h \doteq 0} \\ t'' &= t' + t \end{aligned}\right\} \tag{4}$$

Expanding (2) to the second powers of h, we find

$$t = ha\left\{f'(u) + \frac{h}{2}f''(u)\right\}i + bh\left\{\varphi'(u) + \frac{h}{2}\varphi''(u)\right\}j + ch\left\{\psi'(u) + \frac{h}{2}\psi''(u)\right\}k_{h \doteq 0},$$

$$t' = ha\{f'(u) + \tfrac{3}{2}hf''(u)\}i + bh\{\varphi'(u) + \tfrac{3}{2}h\varphi''(u)\}j + ch\left\{\psi'(u) + \frac{3h}{2}\psi''(u)\right\}k_{h \doteq 0},$$

$$t'' = 2ha\{f(u) + hf''(u)\}i + 2bh\{\varphi'(u) + h\varphi''(u)\}j + 2ch\{\psi'(u) + h\psi''(u)\}k_{h \doteq 0}.$$

$$t \times t = h^3ab\{f'(u)\varphi''(u) - f''(u)\varphi'(u)\}i + h^3bc\{\varphi'(u)\psi''(u) - \varphi''(u)\psi'(u)\}j$$
$$+ h^3ac\{\psi'(u)f''(u) - \psi''(u)f'(u)\}k_{h \doteq 0}.$$

Substituting these values in (3), we find as $h = 0$

$$R^2 = \frac{(t \cdot t)^3}{(t \times t)\cdot(t \times t)}\,.$$

In the case of the helix,

$$r = a\cos ui + a\sin uj + cuk; \qquad t \cdot t = (a^2 + c^2)h^2_{h \doteq 0};$$

and

$$t \times t = h^3\{a^2i + ac\sin uj - ac\cos uk\}_{h \doteq 0}.$$

Hence,

$$R = \frac{a^2 + c^2}{a}\,.$$

NOTE.—The dots over the t's in the denominator of the problem as it appeared in the MONTHLY are misprints, according to a statement of the Proposer. We are publishing Mr. Weisner's solution of the problem as it appeared in print and also Professor Reynolds's solution of the problem as he wished it to appear.—EDITOR OF PROBLEMS AND SOLUTIONS.

322 (Mechanics). Proposed by FRANK R. MORRIS, Glendale, California.

A pole of uniform size and weight throughout its length stands in a vertical position. The height of the pole is h and its weight w. It hinges at the base and falls, passing through a horizontal position. At the moment it reaches the horizontal position, how far from the base is the maximum vertical force tending to break the pole? How great is this force? What is the horizontal force at the same position in the pole?

I. SOLUTION BY C. F. GUMMER, Kingston, Ontario.

Let the rod coincide initially with the y-axis and later with the x-axis. Let θ be its inclination to the latter at time t; P, Q, and N the radial and transverse stresses and the breaking couple at distance r from the origin.

The equations of motion for the element dr are

$$\frac{\partial P}{\partial r}\cos\theta - \frac{\partial Q}{\partial r}\sin\theta = \frac{wr}{gh}\left(\frac{d}{dt}\right)^2\cos\theta, \tag{1}$$

$$\frac{\partial P}{\partial r}\sin\theta + \frac{\partial Q}{\partial r}\cos\theta = \frac{wr}{gh}\left(\frac{d}{dt}\right)^2\sin\theta + \frac{w}{h}, \tag{2}$$

and

$$Q + \frac{\partial N}{\partial r} = 0. \tag{3}$$

The boundary conditions are

$$P = Q = N = 0 \quad \text{when} \quad r = h, \text{ and} \tag{4}$$

$$N = 0 \quad \text{when} \quad r = 0. \tag{5}$$

The initial condition is

6) $$\frac{d\theta}{dt} = 0 \quad \text{when} \quad \theta = \frac{\pi}{2}.$$

By integrating (1) and (2) for r, and using (4), we get

$$P = \frac{w}{2gh}(h^2 - r^2)\left(\frac{d\theta}{dt}\right)^2 - \frac{w}{h}(h - r)\sin\theta,$$

$$Q = -\frac{w}{2gh}(h^2 - r^2)\frac{d^2\theta}{dt^2} - \frac{w}{h}(h - r)\cos\theta,$$

and

$$N = -\int_0^r Qdr = \frac{w}{2gh}\left(h^2r - \frac{r^3}{3}\right)\frac{d^2\theta}{dt^2} + \frac{w}{h}\left(hr - \frac{r^2}{2}\right)\cos\theta.$$

Hence, by (5),

$$2h\frac{d^2\theta}{dt^2} = -3g\cos\theta$$

and hence, by (6),

$$h\left(\frac{d\theta}{dt}\right)^2 = 3g(1 - \sin\theta).$$

It follows that, when $\theta = 0$,

$$P = \frac{3w}{2h^2}(h^2 - r^2), \qquad Q = \frac{w}{4h^2}(h - r)(3r - h), \qquad \text{and} \qquad N = \frac{w}{4h^2}r(h - r)^2.$$

Therefore, the shear, Q, is a maximum for $r = 2h/3$, which makes $P = 5w/6$, $Q = w/12$, and $N = hw/54$.

It may be added that N is a maximum for $r = h/3$, which gives $P = 4w/3$, $Q = 0$, and $N = hw/27$.

This problem is discussed in E. J. Routh's *Elementary Rigid Dynamics*, § 151, 7th edition.

II. Solution by J. B. Reynolds, Lehigh University, South Bethlehem, Pa.

At any instant, let the pole make an angle θ with the vertical and let O be the point about which it is turning. Taking moments about O, we have

$$w\frac{h}{2}\sin\theta = \frac{w}{g}\frac{h^2}{3}\ddot{\theta},$$

or

$$\omega\frac{d\omega}{d\theta} = \ddot{\theta} = \frac{3g}{2h}\sin\theta, \tag{1}$$

whence

$$\omega^2 = \dot{\theta}^2 = \frac{3g}{h}(1 - \cos\theta), \tag{2}$$

ω being the angular velocity of the pole.

When in a horizontal position, let any differential mass of the pole of mass k per unit length be acted upon by a vertical stress force F and a horizontal force T. If this differential of mass is at distance r from O, its accelerations will be $r\ddot{\theta}$ vertically and $r\dot{\theta}^2$ horizontally.

Resolving vertically, we have

$$F + gdm = r\ddot{\theta}dm, \tag{3}$$

in which $dm = kdr$.

Then if M is the moment of the couple tending to break the pole at a point distant u from O, we have

$$M = kg\int_u^h F(r - u) = \int_u^h (r\ddot{\theta} - g)(r - u)dm, \text{ by (3), or by (1) for } \theta = 90^\circ,$$

$$M = kg\int_u^h (3r/2h - 1)(r - u)dr = w/2h^2\int_u^h (3r - 2h)(r - u)dr, \ = w/4h^2\{h^2u + u^3 - 2hu^2\}$$

since $kgh = w$.

For this to be a maximum $dM/du = 0$, or $3u^2 - 4hu + h^2 = (3u - h)(u - h) = 0$; whence $u = 1/3h$, giving the point at which there is the greatest tendency to break. At this point, $M = wh/27$.

Resolving horizontally for a differential mass, we have $T + dT - T = r\dot{\theta}^2dm$, so that by (2) for $\theta = 90^\circ$,

$$T = \frac{3gk}{h}\int_{1/3h}^h rdr = \frac{3w}{h^2}\cdot\frac{r^2}{2}\Big]_{1/3h}^h = \tfrac{4}{3}w.$$